If a player takes risk $x$ on a service, the service is correct with probability $1-x$. The probability to win the rally afterwards, given the service is correct, is $P(x)$. For a player in a certain match, $x_1$, $x_2$, $P(x_1)$ and $P(x_2)$ are known. These are two points in the $x-y$ plane. The question is how to connect these two points. One could draw a straight line. But Geoff and Graham Pollard showed it is likely that the increase in probability of winning the rally decreases when the player takes more risk. They use:

$$P(x) = a + bx + cx^2, \tag{1}$$

where $b > 0$ en $c < 0$. Now we have a problem, since we have to estimate three parameters, but only two point are given. According to Pollard and Pollard, for the avarage player, $c = -0.6$ for men, en $c = -0.3$ for women. Assuming that, $a$ and $b$ can be estimated by:

$$P(x_1) = a + bx_1 + cx_1^2 \tag{2}$$

$$P(x_2) = a + bx_2 + cx_2^2. \tag{3}$$

Now we can also determine the optimal strategy. The probability to win a point is equal to the probability that the first service is correct, and the eally is won afterwards, plus the probability that the first service is incorrect, the second serve is correct, and the rally is won afterwards:

$$P = (1 - x_1)P(x_1) + x_1(1 - x_2)P(x_2). \tag{4}$$

First we can maximize the part regarding the second serve, by setting the first derivative equal to zero:

$$max\{(1 - x_2)(a + bx_2 + cx_2^2)\} \tag{5}$$

$$max\{a + (b - a)x_2 + (c - b)x_2^2 - cx_2^3\} \tag{6}$$

$$-3cx_2^2 + 2(c - b)x_2 + (b - a) = 0 \tag{7}$$

$$x_2 = \frac{(c - b) + \sqrt{(c - b)^2 + 3c(b - a)}}{3c}. \tag{8}$$

Here we use the $abc$-formula:

$$ax^2 + bx + c = 0 \tag{9}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{10}$$

This gives two solutions. Because $b > 0$ en $c < 0$ we know that the $+$ solution is a maximum, and the$-$ solution a minimum. Now we can do the same for $x_1$:

$$max\{(1 - x_1)(a + bx_2 + cx_1^2) + x_1(1 - x_2)P(x_2)\} \tag{11}$$

$$max\{a + (b - a + (1 - x_2)P(x_2))x_1 + (c - b)x_1^2 - cx_1^3\} \tag{12}$$

$$-3cx_1^2 + 2(c - b)x_1 + (b - a + (1 - x_2)P(x_2)) = 0 \tag{13}$$

$$x_1 = \frac{(c - b) + \sqrt{(c - b)^2 + 3c(b - a + (1 - x_2)P(x_2))}}{3c}. \tag{14}$$

This can give strange results, because the probability to win the rally can exceed 1 due to the choice for c. Therefore we assumed $x$ is in between $x_1$ and $x_2$. This also results in less impact of the choice for a quadratic or a straight line on the outcomes. Using the solver in Excel, $P$ can be maximized, under the constraint that $x_1 \geq (x_{1opt}, x_{2opt}) \geq x_2$.